Lecture 1 ***Features of nonlinear ACS. The main types of nonlinearities in systems***

Usage of nonlinear systems origins from the necessity to reflect the real situation more completely. To nonlinear systems belong all the systems that cannot be described by linear differential equations. For nonlinear systems the principle of superposition does not work, i.e. we cannot sum the system reactions on different disturbances. We'll consider only one type of nonlinear systems is investigated. This system is characterized by the following peculiarities: the system can be represent in the form of connection of two parts (fig.6.1): linear part (*LP)* described by a system of simple linear differential equations with constant coefficients, and nonlinear part (*NP)* containing nonlinear element. Nonlinear element is inertialless and its input *y(t)* output *u(t)* values are connected with each other by the equations (6.2). Thus, nonlinearity of these systems is conditioned by nonlinearity of the static characteristic of one of the system elements. Further we'll be presented typical nonlinear elements of ACS (fig.6.3-6.9) and their mathematical description.

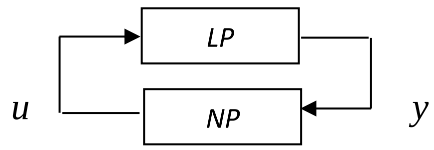

Fig. 6.1. Nonlinear system

Mathematical description of nonlinear ACS can be written as the following:

$$\begin{cases} \dot{x} = Ax + Bu & (6.1) \\ u = f(y), & (6.2) \end{cases}$$

where $x(n \times 1)$ is a state vector; $u(m \times 1)$ is a control vector or a vector of input coordinates; $y(r \times 1)$ is an output coordinates vector. In this book we consider subclass of nonlinear systems having only one input and one output.

In fig.6.2 there are presented characteristics of nonlinear systems where the linear and nonlinear parts are shown clearly.

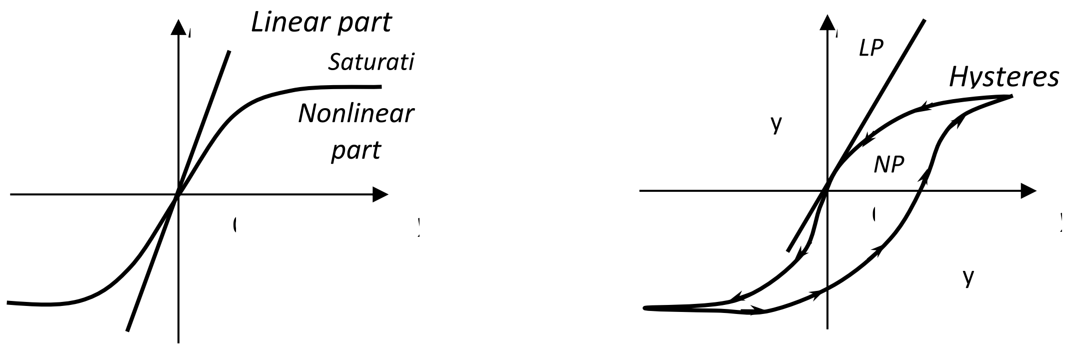

Fig 6.2. Examples of nonlinear ACS characteristics

1.1. The main types of nonlinearity of ACS

In practice we usually meet elements, characteristics of which are partly linear or which approximate to part-linear. Let us consider statistic characteristics of typical nonlinear elements of ACS and their mathematical description as:

$$u = f(y).$$

Mathematical description (fig. 6.3):
$u = M \operatorname{sign} y;$
M – *const*, $M > 0$.

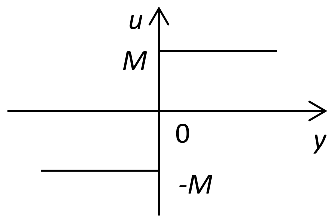

Fig. 6.3. Ideal two-position relay

Mathematical description (fig. 6.4):

$$u = \begin{cases} +M, & y > a \\ 0, & -a \le y \le a \\ -M, & y < -a \end{cases};$$

where a, M – *const.*

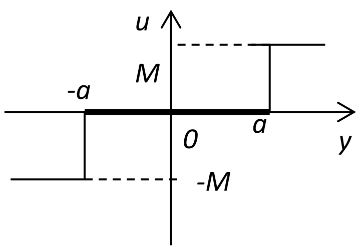

Fig.6.4. Three-position relay with insusceptibility

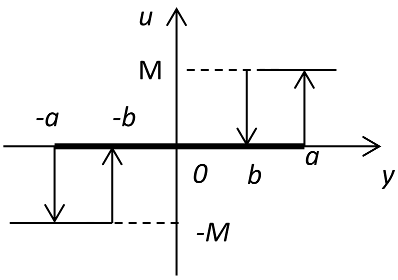

Fig.6.5. Three-position relay with insusceptibility with a hysteresis

Mathematical description (fig. 6.5):

$$\frac{dy}{dt} > 0 \qquad\qquad \frac{dy}{dt} < 0$$

$$u = \begin{cases} +M, & y > a \\ 0, & -b \le y \le a \\ -M, & y < -b \end{cases} \qquad\qquad u = \begin{cases} +M, & y > b \\ 0, & -a \le y \le b \\ -M, & y \le -a \end{cases}$$

where *a, b, M – const.*

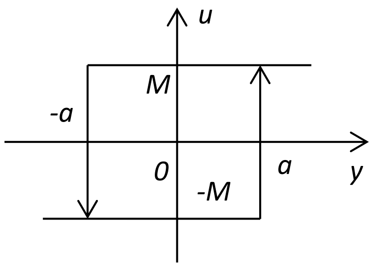

Fig. 6.6. Two-position relay with hysteresis

Mathematical description (fig. 6.6):

At $\frac{dy}{dt} > 0 \quad u = \begin{cases} +M, & y \ge a \\ -M, & y < a \end{cases}$; at $\frac{dy}{dt} < 0 \quad u = \begin{cases} +M, & y \ge -a \\ -M, & y < -a \end{cases}$,

where a and M are const.

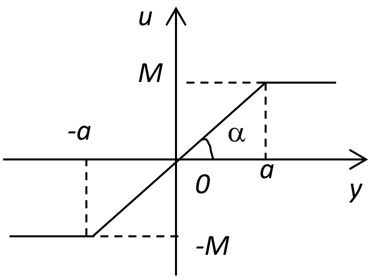

Fig. 6.7. Relay with saturation

Mathematic description (fig. 6.7):

$$u = \begin{cases} +M, & y \geq a \\ ky, & -a < y < a \\ -M, & y \leq -a \end{cases},$$

where $k = tg\,\alpha = M/a$, *"k"* is slope coefficient; α, *a* and *M* are const.

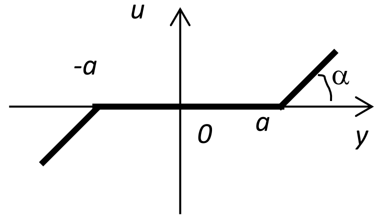

Fig. 6.8. The relay with insusceptibility zone without saturation

Mathematical description (fig. 6.8):

$$u = \begin{cases} k(y-a), & y \geq a \\ ky, & -a < y < a \\ k(y+a), & y \leq -a \end{cases},$$

where $k = tg\,\alpha$, α and *"a"* are const.

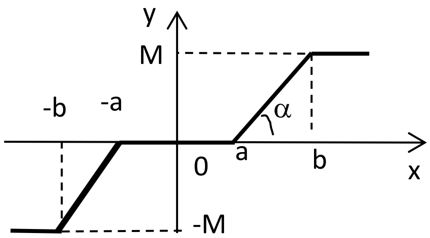

Fig. 6.9. The relay with saturation and with insusceptibility zone

Mathematical description (fig. 6.9):

$$u = \begin{cases} +M, & y > b \\ k(y-a), & a < y < b \\ 0, & -a < y < a \\ k(y+a), & -b < y < -a \\ -M, & y < -b \end{cases},$$

where $k = tg\,\alpha$; *a*, *b* and *M* are const.

1.2 Features of nonlinear ACS

1. We will not apply the principle of superposition to nonlinear systems.

2. Surely it is necessary to allocate the linear part (*LP*) of a system and the nonlinear part (*NP*).

3. Here are presented the most widespread types of nonlinear characteristics (*nonlinear element*), which are used in ACS.

It is necessary to mark, that nonlinear ACS can have:

– ambiguous characteristics;
– several special point-to-points of stationary;
– auto-oscillations.

Auto-oscillations are cyclic motion of nonlinear systems, parameters of which do not depend on the primary conditions.

Nowadays there are developed some exact and approximate methods of nonlinear ACS investigations. In this discipline the following exact methods of ACS investigations we'll be considered:

– phase plane method, suggested by A. A. Andronov (А.А. Андронов);
– the second (direct) method of A. M. Lyapunov (А.М. Ляпунов).

Besides, we will consider the method of a research of absolute stability (stability "in general") nonlinear ACS offered the Romanian scientist by V.M. Popov.